

# Disability Tables for the Brazilian Insurance Market

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## Resumo

This paper describes the construction of disability tables, for males and females, for the insured population in Brazil. The tables were constructed based on data of the years 2004 to 2009 collected from 13 insurance corporations (corresponding to about 80% of the Brazilian insured market). The database was checked for inconsistencies with the official disability data of the Ministry of Social Security (CNIS and SISOBI). The project's goal was to create a credible alternative to tables used by the Brazilian insurance companies, not always representative of the present population, such as the tables of Alvaro Vindas and IAPB-57.

**Keywords:** Disability tables; Insurance; Social Security; Logistic Model.

## 1. Introduction

The present project is one of outcomes of larger project, starting in 2007, involving the Applied Mathematics Lab of UFRJ (LabMA/UFRJ) and sponsored by the National Federation of Private Pension Insurance Companies (FENAPREVI). The government regulating agency (SUSEP – Superintendência de Seguros Privados) supervised the project, with the intention of later adopting these tables for the Brazilian insurance market.

LabMA/UFRJ constructed the mortality tables *BR-EMSsb* and *BR-EMSmt* based on the Brazilian insured population for the years 2004, 2005 and 2006.

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Essa pesquisa teve o apoio da Fundação de Amparo à Pesquisa do Estado do Rio de Janeiro (FAPERJ).

These tables are standard in Brazil, according to SUSEP directive nº 402, of March 18<sup>th</sup>, 2010.

There are no disability tables in use in Brazil which reflect the recent experience of this population. Therefore, insurance companies tend to use either foreign tables, such as Álvaro Vindas [6], built for the Costa Rican population, for 1957, or, for instance, IAPB-57, constructed in 1957 by the late *Instituto de Aposentadoria e Pensões dos Bancários*. In particular, Álvaro Vindas disability table has a central role in the actuarial work of insurance companies in Brazil.

The present tables, named *BR-EMSeinv* (*BR* stand for Brazil; *EMS* – Experiência do Mercado Segurador – stand for Insurance Market Experience; *inv* – entrada em invalidez – stand for disability), fill the gap in terms of population and time.

The present paper is organized as follows. In Section 2, the database is analyzed from the viewpoints of construction starting with the insurance companies data and later concatenation with the government's database of the Social Security Ministry. In Section 3 describes the methodology for the disability rates graduation. The main conclusions are in Section 4, while Table 7 depicts the disability tables, for males (*BR-EMSeinv-m*) and females (*BR-EMSeinv-f*).

## 2. The Database and Table Construction Viability

In this section we analyze the data collected from insurance companies and present the methodology to determine the disability crude rates.

Initially, we study the table construction viability. This is demonstrated by the similarity between the curves of crude disability rates and the usual pattern of the disability tables. Moreover, one expects a distinction between the males and females rates.

LabMA/UFRJ received from the insurance companies data of the years 2004 to 2009, concerning the population insured for disability, in terms of exposure and disability events. This is the data the companies are required to inform SUSEP annually.

The number of disabilities informed in the data was much lower than expected

from the usual disability tables. This happened because the disability coverage is usually a subsidiary coverage among other more important protections, and the disability events are not reported in many instances. Therefore, LabMA/UFRJ searched for the correction of the original data consulting the government databases National Base of Social Data (*Cadastro Nacional de Informações Sociais – CNIS*) and the Death Registration System (*Sistema de Controle de Óbitos – SISOB*).

As a result of the above process, the following tables, for males and females, show, for each calendar year, the observed disability numbers (and percentages) and the expected disability numbers (and percentages) calculated from the tables of Álvaro Vindas, IABP-57 weak, and disability table for the whole Brazilian population (RGPS). Note that the correction process with the government’s databases resulted in multiplying by fifteen times the original numbers supplied by the companies. Of course this correction was fundamental to the construction of the tables.

LabMA/UFRJ determined expositions to risk, number of disabilities for each age, sex, and calendar year. As a result one determined, for males and females, the crude disability rates

$$i_x = \frac{\sum_{t=2004}^{2009} N_{xt}}{\sum_{t=2004}^{2009} e_{xt}}$$

where  $N_{xt}$  is the number of disabilities at age  $x$  and year  $t$  and  $e_{xt}$  is the exposition for age  $x$  and year  $t$ . The crude disability curves are shown in the following graph, according to sex and compared to well-known disability curves.

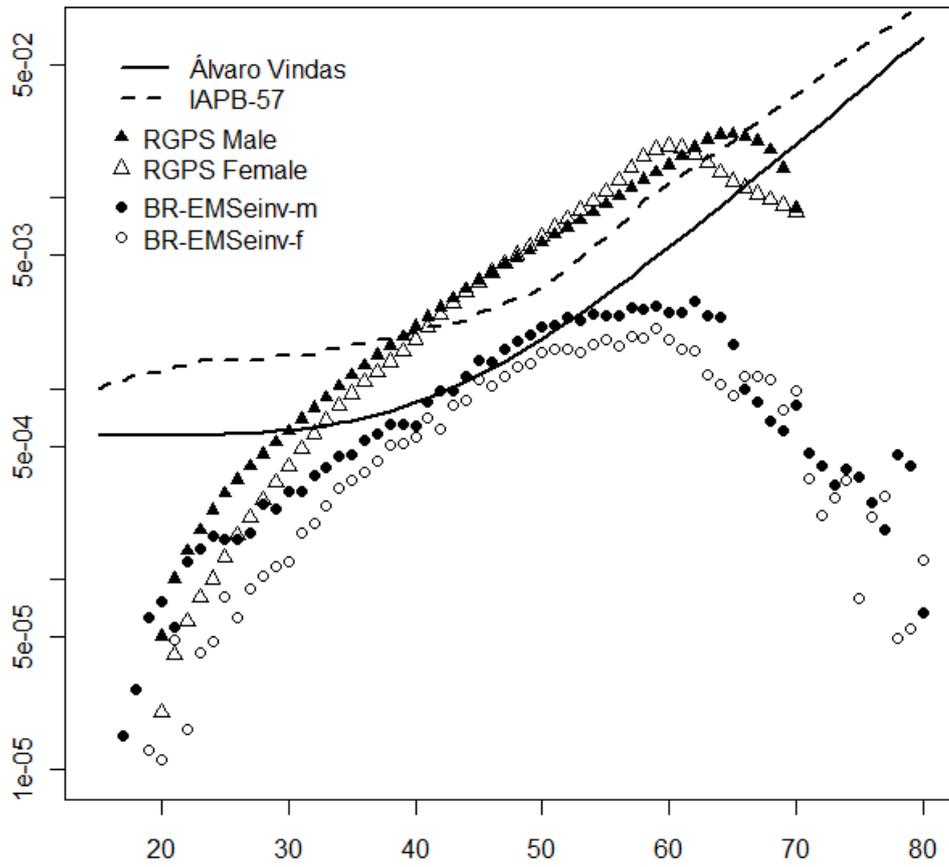


Figura 1: Crude mortality rates by age (on a logarithmic scale)

The graph shows a distinct separation between the curves for males and females. Moreover, these curves show the expected characteristics usually found in disability rates, up to approximately 60 years of age. After that, there is clearly a sub enumeration of disabilities due to the fact that, in Brazil, retired people don't have an incentive to report a later disability. This phenomenon is also reported for the Brazilian population as a whole, as can be seen in the RGPS tables.

### 3. Graduation of Disability Crude Rates

The number of disabilities can be seen as a random phenomenon with a known distribution assuming that all individuals with the same age come into disability independently and with the same probability, thus being a *Binomial*( $e_x, i_x$ ) distribution with the probability parameter  $i_x$  unknown or a *Poisson*( $\lambda_x$ ) distribution with mean  $\lambda_x = e_x \mu_x$ , where  $\mu_x$  is the force of disability at age  $x$  assumed unknown and constant in each age bracket.

Assuming further that disability crude rates are smooth and monotonically increasing, the usual way to graduate these rates is through some generalized linear model [3], using either the Poisson distribution (if we are graduating as a function of the force of disability) or the Binomial distribution (if we are graduating as a function of the probability of disability). In the present work, the logistic model was chosen, that is, the Binomial model with its canonical link function, the logit:

$$\text{logit } i_x = \log \left( \frac{i_x}{1 - i_x} \right) = \alpha + \beta x, \quad \alpha, \beta \in \mathbb{R} \quad (1)$$

A particularity already mentioned about these data is that from about 60 years onwards, there is a significant underenumeration of disability occurrences. As it is not possible to know the intensity of this underenumeration throughout the ages, we choose to fit the model only up to age 50, extrapolating from then onwards.

Table 1 shows the results using the statistical software R (R Development Core Team, 2011).

Tabela 1: Results for both sexes

	$\hat{\alpha}$	$\hat{\beta}$	Deviance	d.f.	AIC
Males	-11.3513	0.1043	35.07	32	256.71
Females	-12.2829	0.11845	63.14	30	250.71

One notes that the residual deviance is rather high for females. Visually, one verifies that the adjustment was not adequate for ages below 30. McCullagh e Nelder (1989) provides a test for the usefulness of a power transformation to the explanatory variable. In case the null hypothesis is rejected, the model may be replaced by

$$\text{logit } i_x = \alpha + \beta x^\lambda \quad (2)$$

For females, p-value for this test was approximately  $10^{-10}$ . Therefore, we modified the model accordingly and estimated the additional parameter  $\lambda$  in order to minimize the deviance function. The results are in the table below.

Tabela 2: First model adjust for the female population

	$\hat{\alpha}$	$\hat{\beta}$	Deviance	d.f.	AIC	$\hat{\lambda}$
Females	5.2333	-46.6243	20.78	30	208.35	-0.3531

However, for the extrapolation part, it was used model (1) readjusted considering ages between 34 and 50 years (see Table 3). Therefore, the projection of the rates is linear on the logistic scale and is based solely on the trend that these rates have in adulthood. As model (2) adjusted (responsible for the first part of the curve, fitted to the data) and model (1) readjusted (responsible for the second part of the curve, the projection) almost coincide in the age span from 34 and 50 years, the transition between one curve and another is smooth and occurs at age 48.

Tabela 3: First model readjusted for the female population

	$\hat{\alpha}$	$\hat{\beta}$	Deviance	d.f.	AIC
Females	-11.5104	0.1013	14.93	15	137.68

It is worth noting that, in all cases, the test for overdispersion was not significant and the dispersion parameter was taken to be 1. Moreover, confidence intervals are very narrow due to the high exposure at every age. Table 4 shows the minimum and maximum exposure for each gender until the age 50.

Tabela 4: Minimum and Maximum Exposure

Males		Females	
Age	Exposure	Age	Exposure
17	66.807	18	66.962
41	380.430	34	265.921

Figure 2 presents the final disability tables by gender. As a comparison guide, tables Álvaro Vindas and RGPS are also plotted.

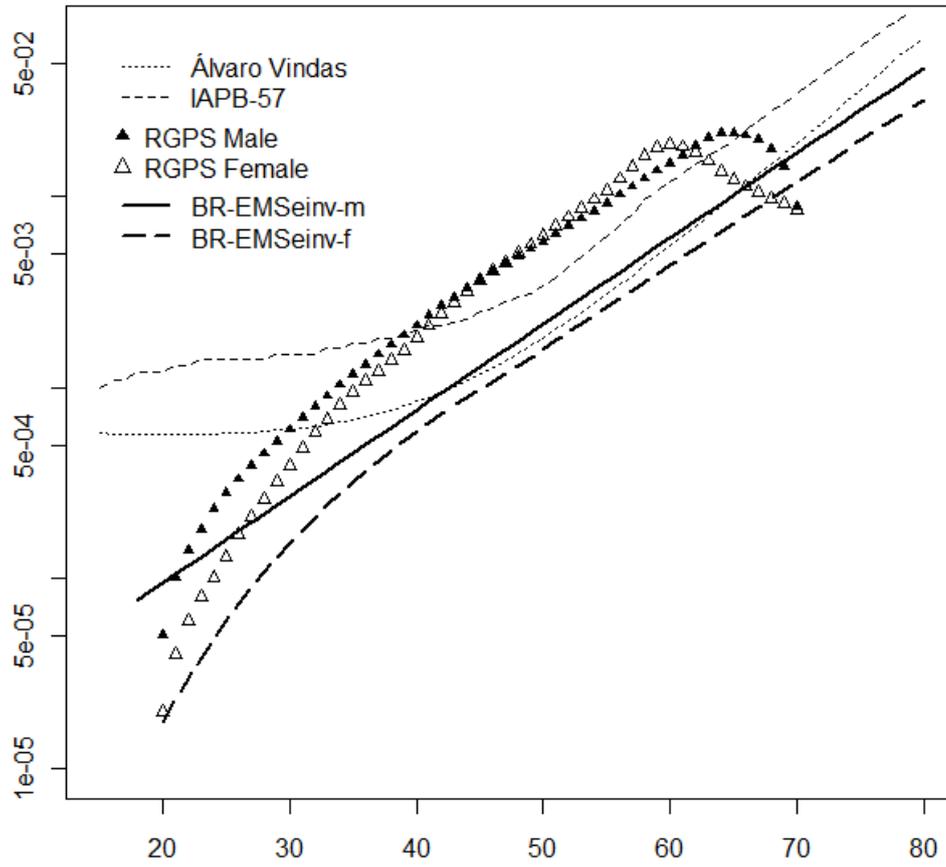


Figura 2: Estimated probabilities of disability by age (on a logarithmic scale)

For the projection part of the curve, the estimates of the parameter  $\beta$  (which can be interpreted as a growth rate) are similar for both genders, so the final curves are almost parallel between ages 50 and 80.

## 4. Conclusion

Each disability curve has one branch until the age of 50 year. The adjustment used the logistic GLM. From this point on, the recommended model for insurance purposes is the extrapolation of the initial logistic equation. Table 7 presents the proposed disability tables, BR-EMSeinv, for males and females.

We note that the Álvaro Vindas disability table, each applies to both sexes, lies above the BR-EMSeinv for all ages except from 43 to 65 years of age. In this age interval the BR-EMSeinv is, on average, 12% above Álvaro Vindas disability table. Considering all ages, BR-EMSeinv is, on average, 26% below Álvaro Vindas disability table, for males, and 48% below for females.

We recommend the introduction of the *BR-EMSeinv* disability tables in the actuarial work in Brazil, not only in the insurance industry, but also in all types of private pension funds.

## Referências

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Tabela 5: BR-EMSeinv-m e BR-EMSeinv-f

Idade	Homens	Mulheres	Idade	Homens	Mulheres
15	0.000056	0.000003	53	0.002943	0.002152
16	0.000062	0.000005	54	0.003266	0.002381
17	0.000069	0.000007	55	0.003623	0.002635
18	0.000077	0.000009	56	0.004020	0.002915
19	0.000085	0.000013	57	0.004460	0.003225
20	0.000095	0.000017	58	0.004947	0.003567
21	0.000105	0.000023	59	0.005488	0.003946
22	0.000117	0.000030	60	0.006087	0.004366
23	0.000129	0.000038	61	0.006752	0.004829
24	0.000144	0.000048	62	0.007488	0.005341
25	0.000159	0.000060	63	0.008304	0.005908
26	0.000177	0.000073	64	0.009209	0.006534
27	0.000196	0.000089	65	0.010210	0.007225
28	0.000218	0.000107	66	0.011320	0.007990
29	0.000242	0.000128	67	0.012548	0.008835
30	0.000268	0.000151	68	0.013908	0.009768
31	0.000298	0.000178	69	0.015413	0.010798
32	0.000330	0.000208	70	0.017077	0.011936
33	0.000367	0.000241	71	0.018919	0.013192
34	0.000407	0.000277	72	0.020954	0.014579
35	0.000452	0.000318	73	0.023203	0.016109
36	0.000501	0.000363	74	0.025688	0.017797
37	0.000556	0.000412	75	0.028430	0.019657
38	0.000617	0.000465	76	0.031457	0.021709
39	0.000685	0.000523	77	0.034793	0.023968
40	0.000761	0.000586	78	0.038470	0.026457
41	0.000844	0.000654	79	0.042517	0.029197
42	0.000937	0.000728	80	0.046970	0.032211
43	0.001040	0.000807	81	0.051864	0.035524
44	0.001154	0.000892	82	0.057238	0.039165
45	0.001280	0.000982	83	0.063131	0.043162
46	0.001421	0.001079	84	0.069586	0.047546
47	0.001577	0.001182	85	0.076647	0.052352
48	0.001750	0.001298	86	0.084359	0.057614
49	0.001942	0.001436	87	0.092770	0.063370
50	0.002154	0.001589	88	0.101925	0.069658
51	0.002391	0.001758	89	0.111873	0.076519
52	0.002653	0.001945	90	0.122659	0.083995